MODEL PREDICTIVE CONTROL

STATE OF THE ART AND POSSIBLE OPPORTUNITIES FOR LIFE-SUPPORT SYSTEMS

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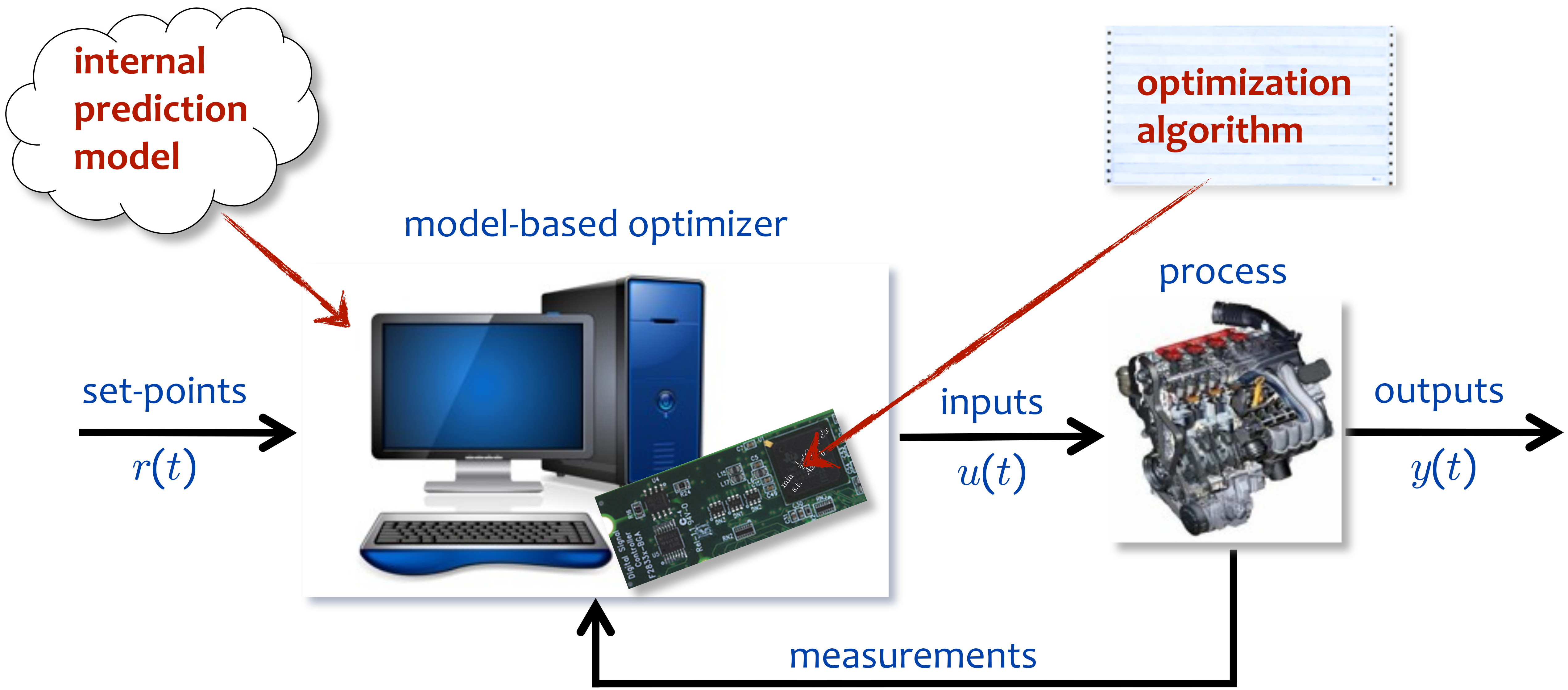
ODYS Advanced Controls & Optimization

Melissa Workshop - Lausanne, June 8-9, 2016

MODEL PREDICTIVE CONTROL (MPC)



simplified　　　　　　　　　　　　　　　likely

Use a dynamical ***model*** *of the process to* ***predict*** *its future evolution and choose the "best"* ***control*** *action*

MODEL PREDICTIVE CONTROL (MPC)

- At each time t, find the best control sequence over a future horizon of N steps

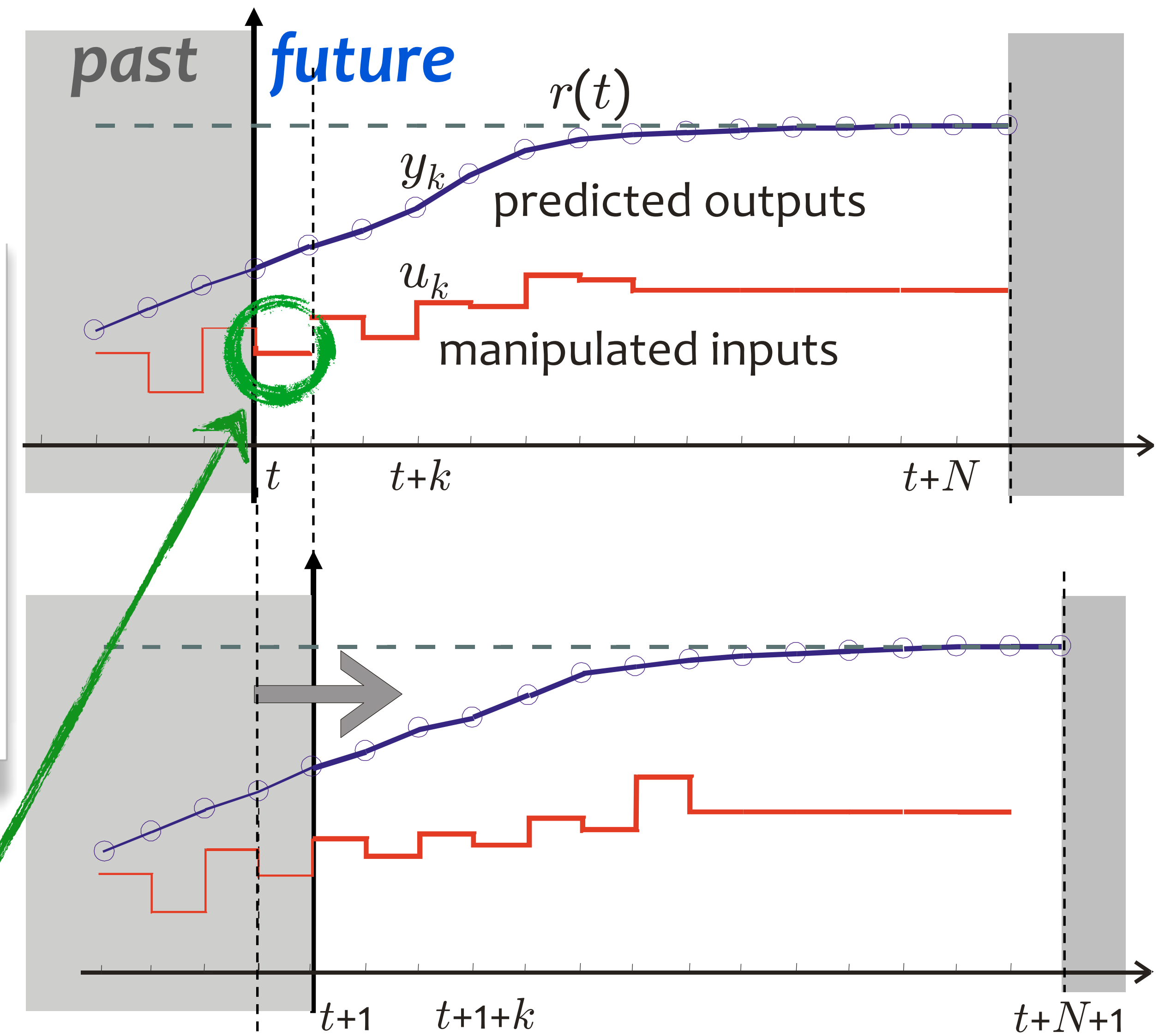
penalty on tracking error

penalty on actuation

$$\begin{aligned} \min \quad & \sum_{k=0}^{N-1} \|W^y(y_k - r(t))\|^2 + \|W^u(u_k - u^{\text{ref}}(t))\|^2 \\ \text{s.t.} \quad & x_{k+1} = f(x_k, u_k, t) \\ & y_k = g(x_k, u_k, t) \\ & \text{constraints on } u_k, y_k \\ & x_0 = x(t) \end{aligned}$$

← feedback !

numerical optimization problem



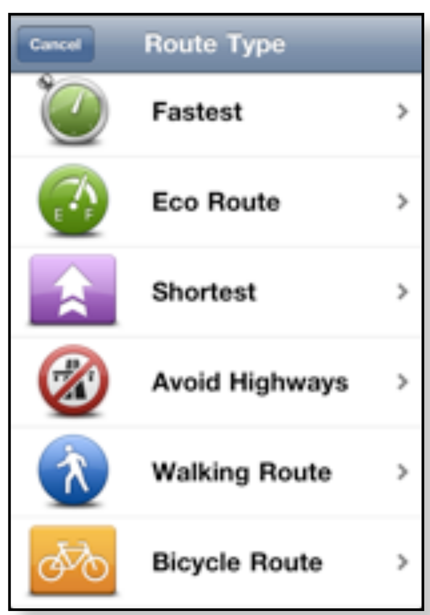
- Only apply the first optimal move $u(t)$, throw the rest of the sequence away

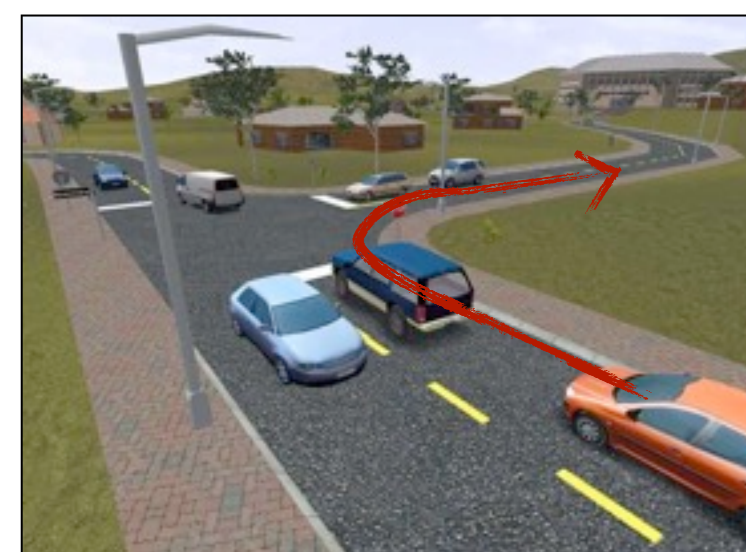
- At time $t+1$: **Get new measurements**, **repeat the optimization**. And so on ...

THE MPC CONCEPT

- MPC is like **playing chess** !


- On-line re-planning while driving:





MPC IN INDUSTRY

- The idea of using MPC in the process industries dates back to the sixties

Discrete Dynamic Optimization Applied to On-Line Optimal Control

MARSHALL D. RAFAL and WILLIAM F. STEVENS

(Rafal, Stevens, *AiChE Journal*, 1968)

Esso

- MPC used in process industries since the 80's

Area	Aspen Technology	Honeywell Hi-Spec	Adersa[b]	Invensys	SGS[c]	Total
Refining	1200	480	280	25		1985
Petrochemicals	450	80	—	20		550
Chemicals	100	20	3	21		144
Pulp and paper	18	50	—	—		68
Air & Gas	—	10	—	—		10
Utility	—	10	—	4		14
Mining/Metallurgy	8	6	7	16		37
Food Processing	—	—	41	10		51
Polymer	17	—	—	—		17
Furnaces	—	—	42	3		45
Aerospace/Defense	—	—	13	—		13
Automotive	—	—	7	—		7
Unclassified	40	40	1045	26	450	1601
Total	1833	696	1438	125	450	4542
First App.	DMC:1985 IDCOM-M:1987 OPC:1987	PCT:1984 RMPCT:1991	IDCOM:1973 HIECON:1986	1984	1985	
Largest App.	603 × 283	225 × 85	—	31 × 12	—	

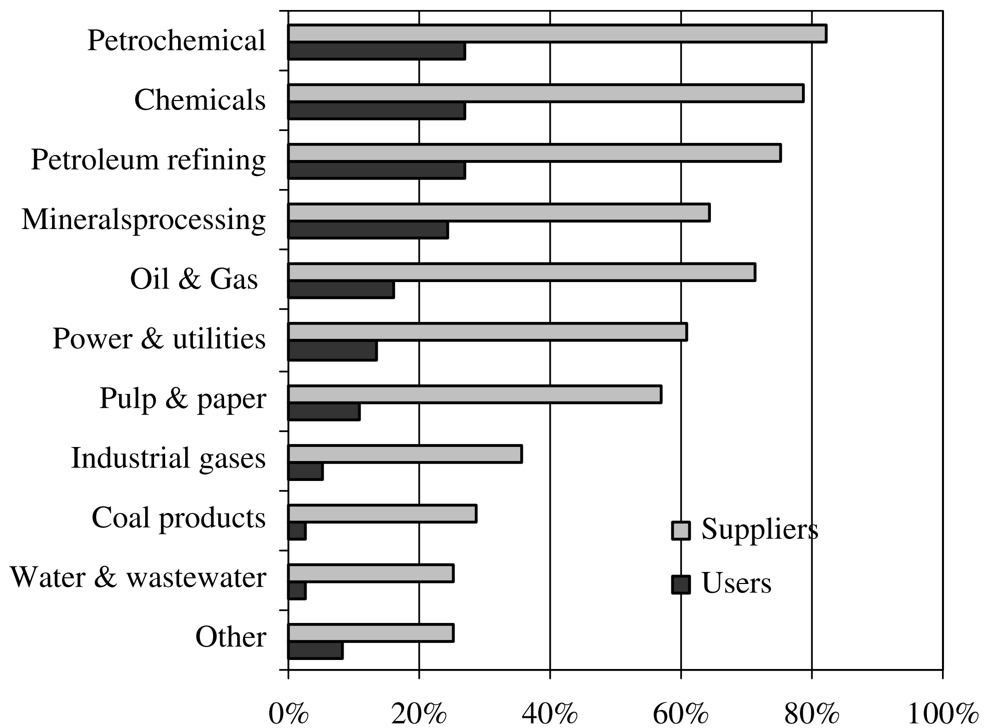
(snapshot survey conducted in mid-1999)

(Qin, Badgewell, 2003)

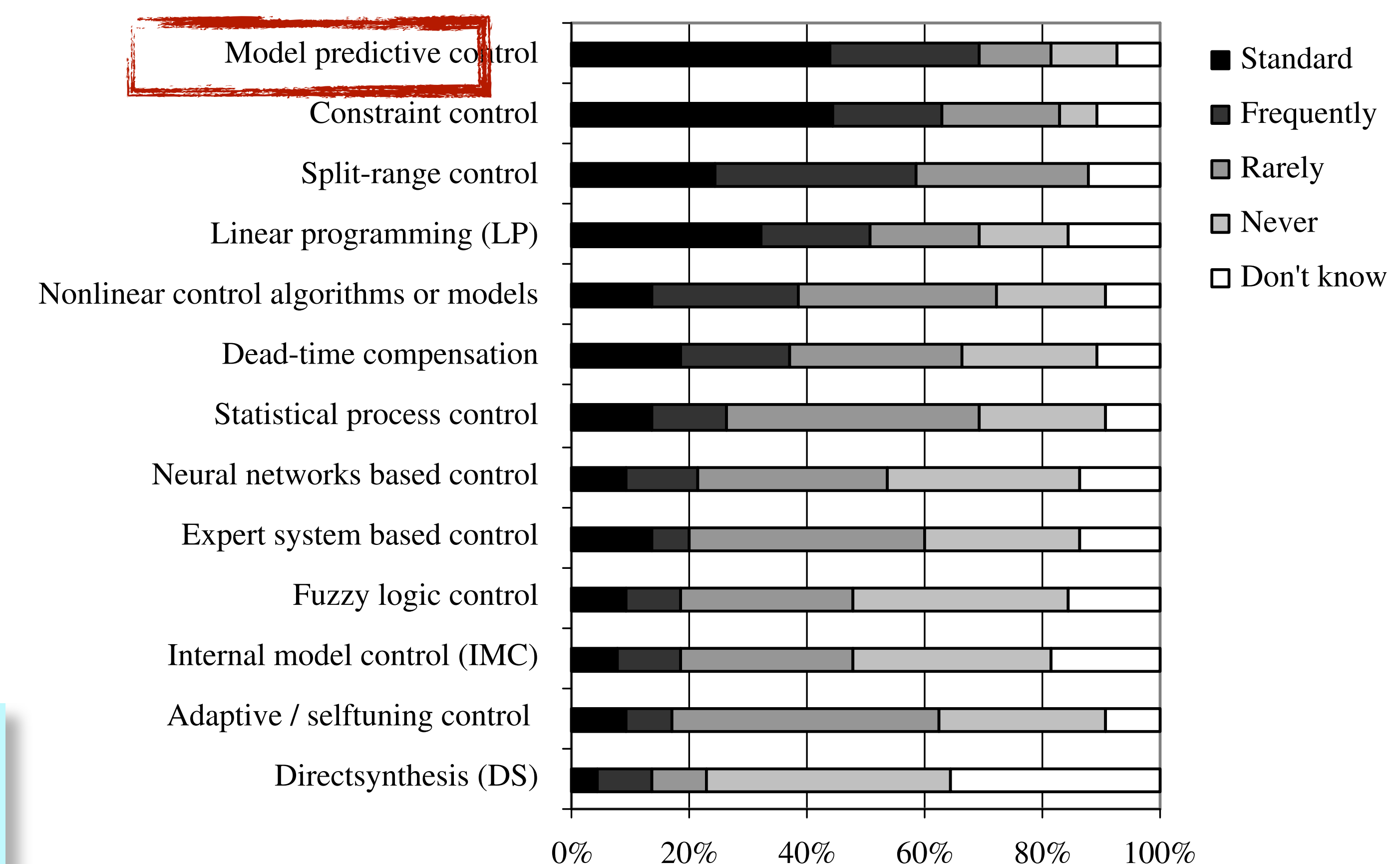
MPC IN INDUSTRY

(Bauer & Craig, 2008)

- Economic assessment of Advanced Process Control (APC)



participants of APC survey by industry (worldwide)

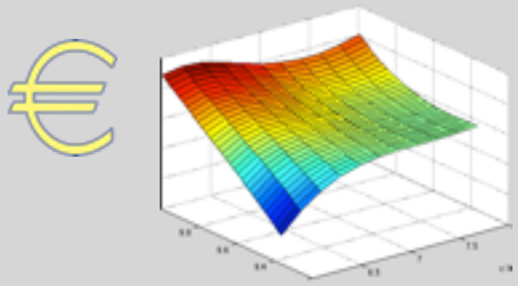


Industrial use of APC methods: survey results

MPC is the *de-facto* standard for advanced control in the process industry.

TYPICAL USE OF MPC

static optimizer

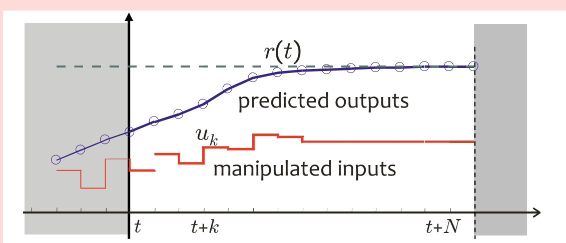


Steady-state optimization

- (economic) set-point optimization
- static I/O model

output set points and
corresponding optimal actuator set points

dynamic optimizer



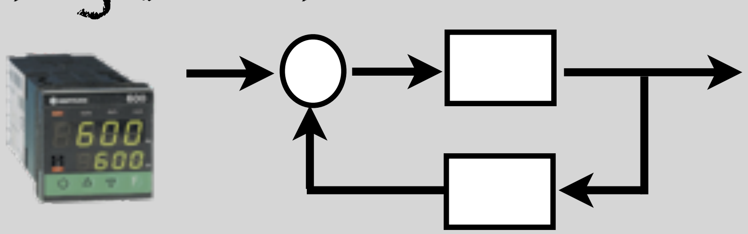
MPC

- coordinate multiple inputs
- performance optim.
- constraints handling

measurements

set points to actuators

regulators

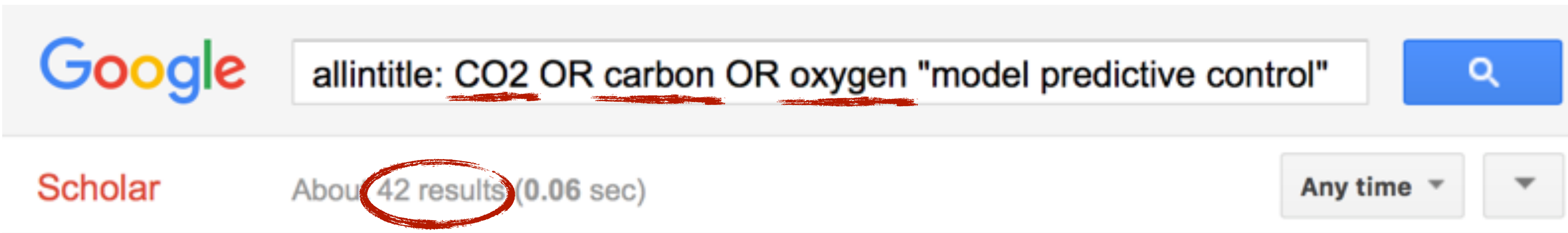


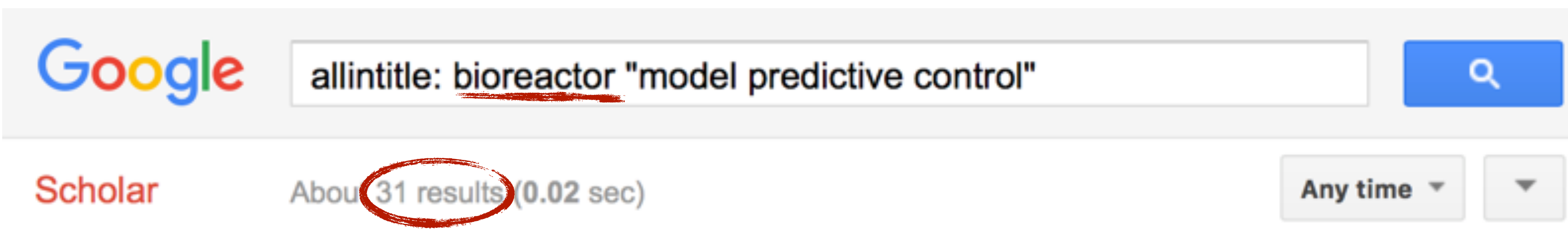
low-level controllers

- fast-sampling
- single-loop

MPC FOR LIFE SUPPORT SYSTEMS

- Quick literature search:

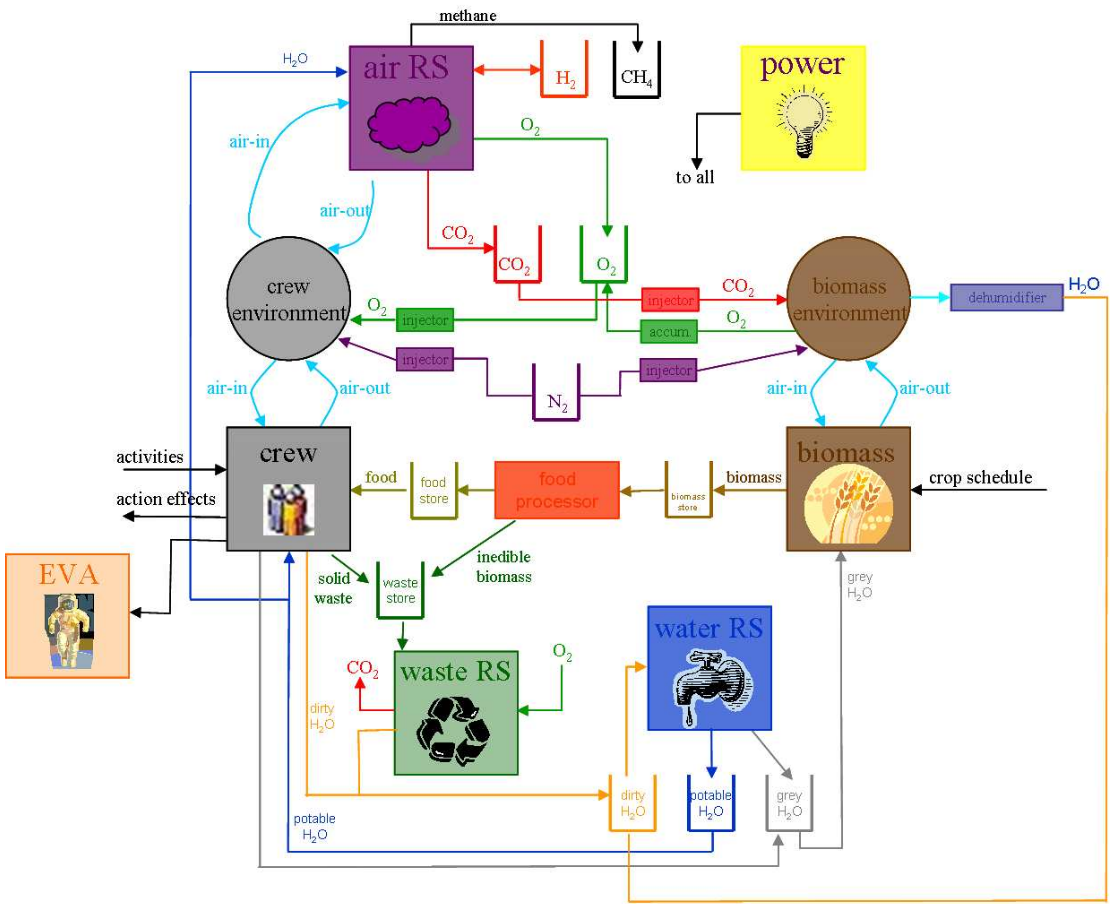




- Sherpa Engineering proposed MPC schemes within MELiSSA

MPC FOR LIFE SUPPORT SYSTEMS

- NASA's environmental control and life support system



- CELSS (Controlled Ecological Life Support System) (Auslander, 1982)

- Honeywell proposed a Nonlinear MPC scheme for Variable Configuration CO2 Removal (VCCR) (Subramanian & Lamba, 2005)

AEROSPACE APPLICATIONS OF MPC

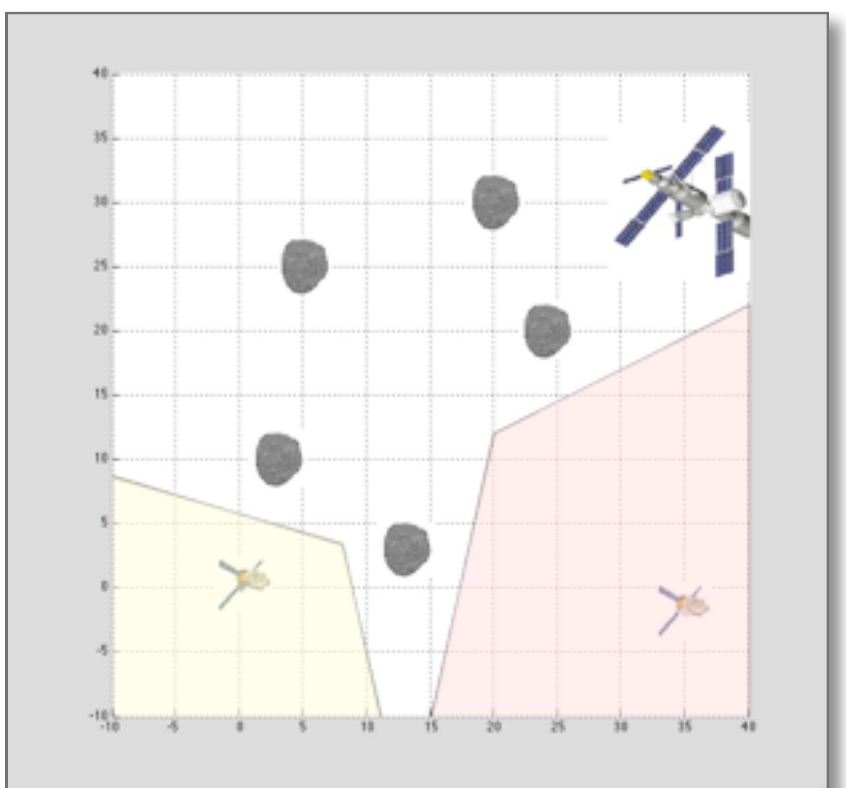
- MPC capabilities explored in new space applications
- New MATLAB MPC Toolboxes developed (**MPCTOOL** and **MPCSofT**)

(Bemporad, 2010) (Bemporad, 2012)

powered descent

(Pascucci, Bennani, Bemporad, 2016)

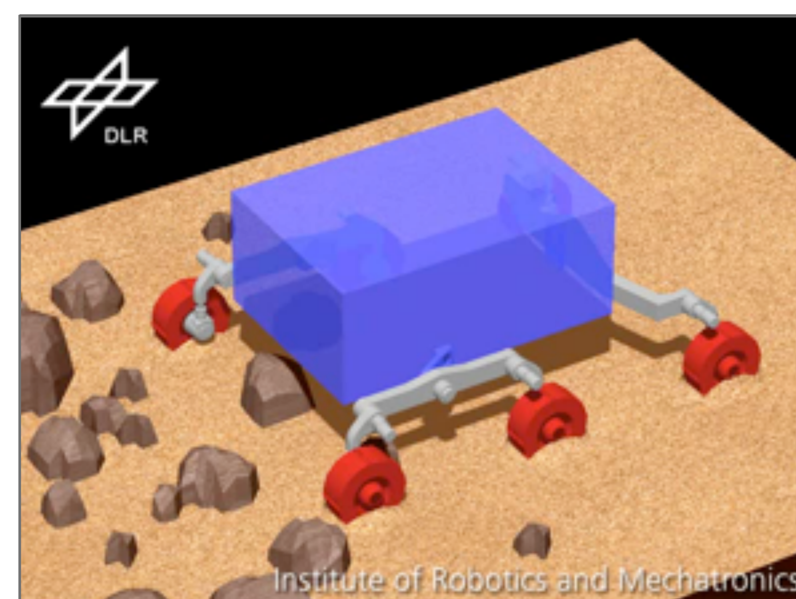
cooperating UAVs



(Bemporad, Rocchi, 2011)

planetary rover

(Krenn et. al., 2012)



AUTOMOTIVE APPLICATIONS OF MPC

Bemporad, Bernardini, Borrelli, Cimini, Di Cairano, Esen, Giorgetti, Graf-Plessen, Hrovat, Kolmanovsky, Levijoki, Ripaccioli, Trimboli, Tseng, Yanakiev, … (2001-2016)

Powertrain

- direct-inj. engine control
- A/F ratio control
- magnetic actuators
- robotized gearbox
- power MGT in HEVs
- cabin heat control in HEVs
- electrical motors

Vehicle dynamics

- traction control
- active steering
- semiactive suspensions
- autonomous driving

Ford Motor Company

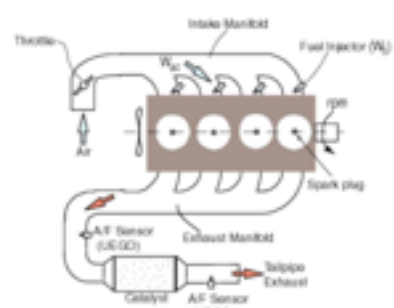
Jaguar

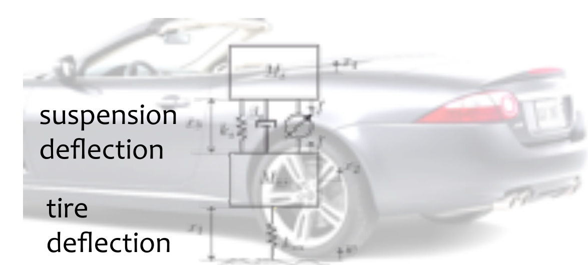
DENSO Automotive

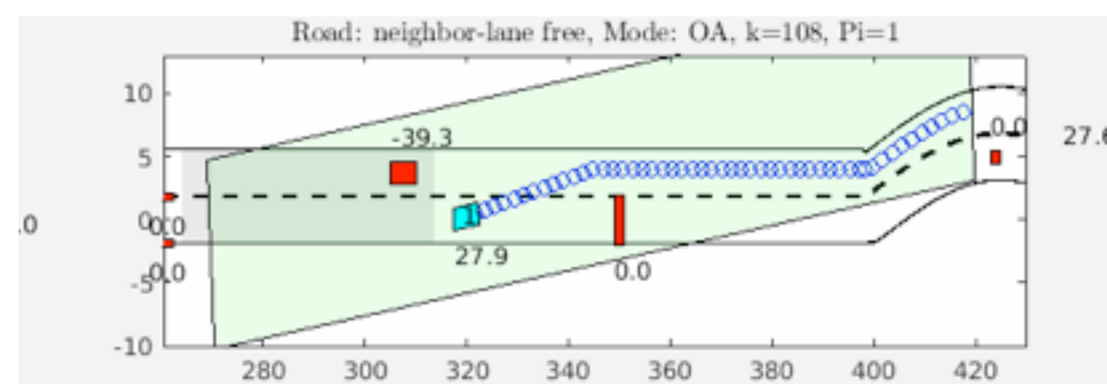
FIAT

General Motors

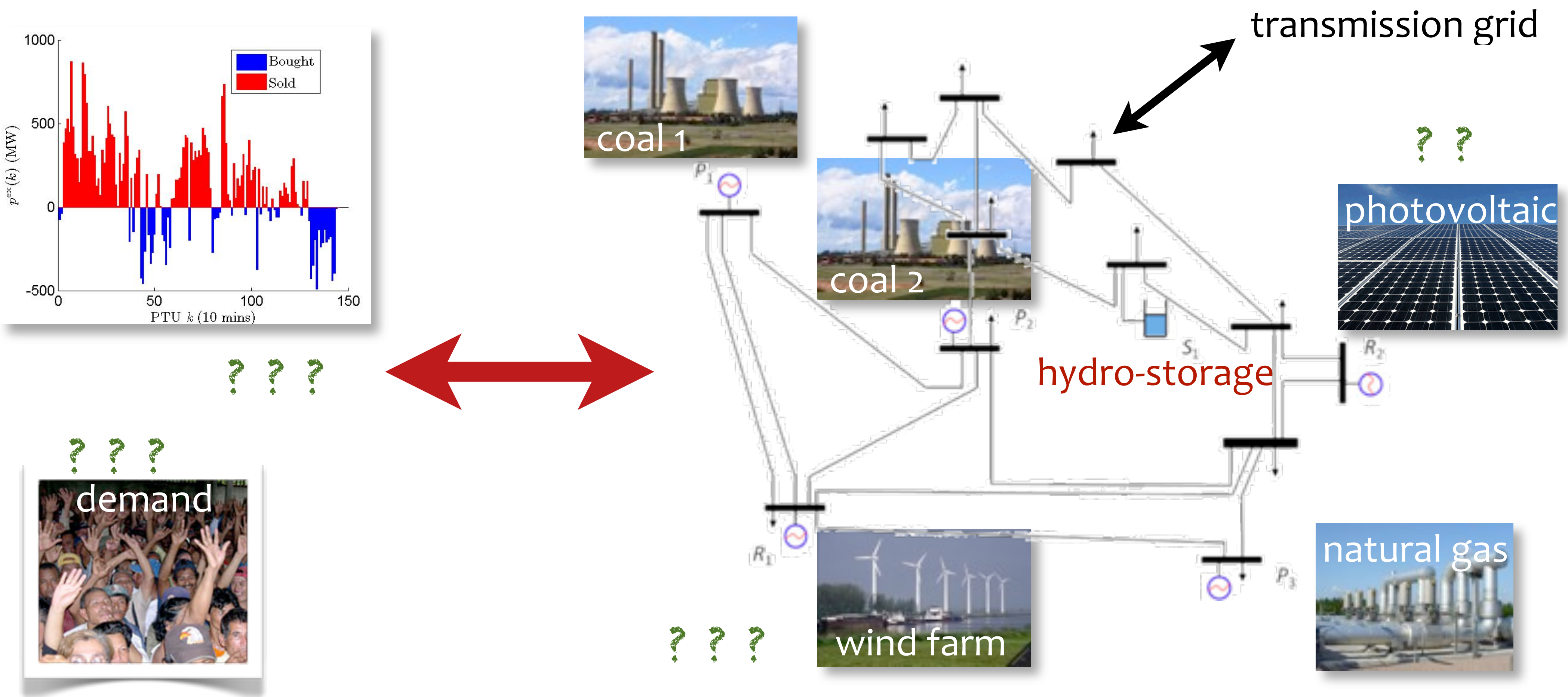
ODYS Advanced Controls & Optimization







MPC FOR SMART ELECTRICITY GRIDS



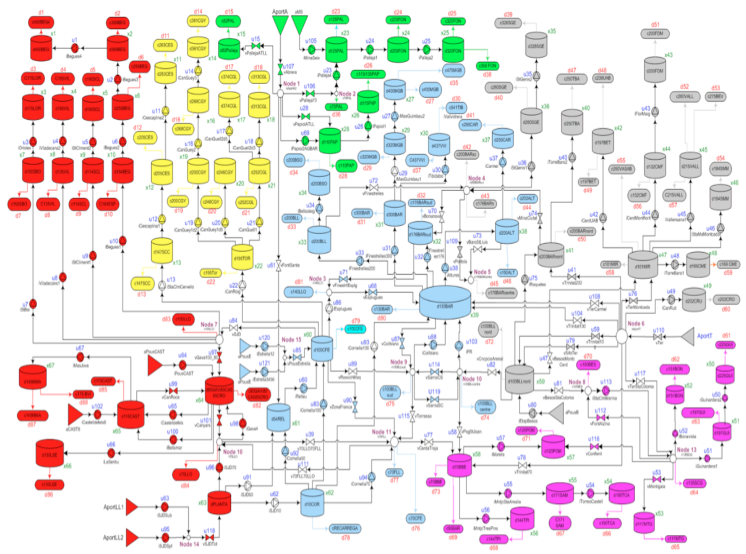
Dispatch power in smart distribution grids, **trade energy** on energy markets

Challenges: account for **dynamics**, network **topology**, physical **constraints**, and **stochasticity** (of renewable energy, demand, electricity prices)

FP7-ICT project "**E-PRICE - Price-based Control of Electrical Power Systems**" (2010-2013)

MPC FOR MANAGEMENT OF DRINKING WATER NETWORKS

Drinking water network of Barcelona:
81 tanks,
64 valves
180 pumps.

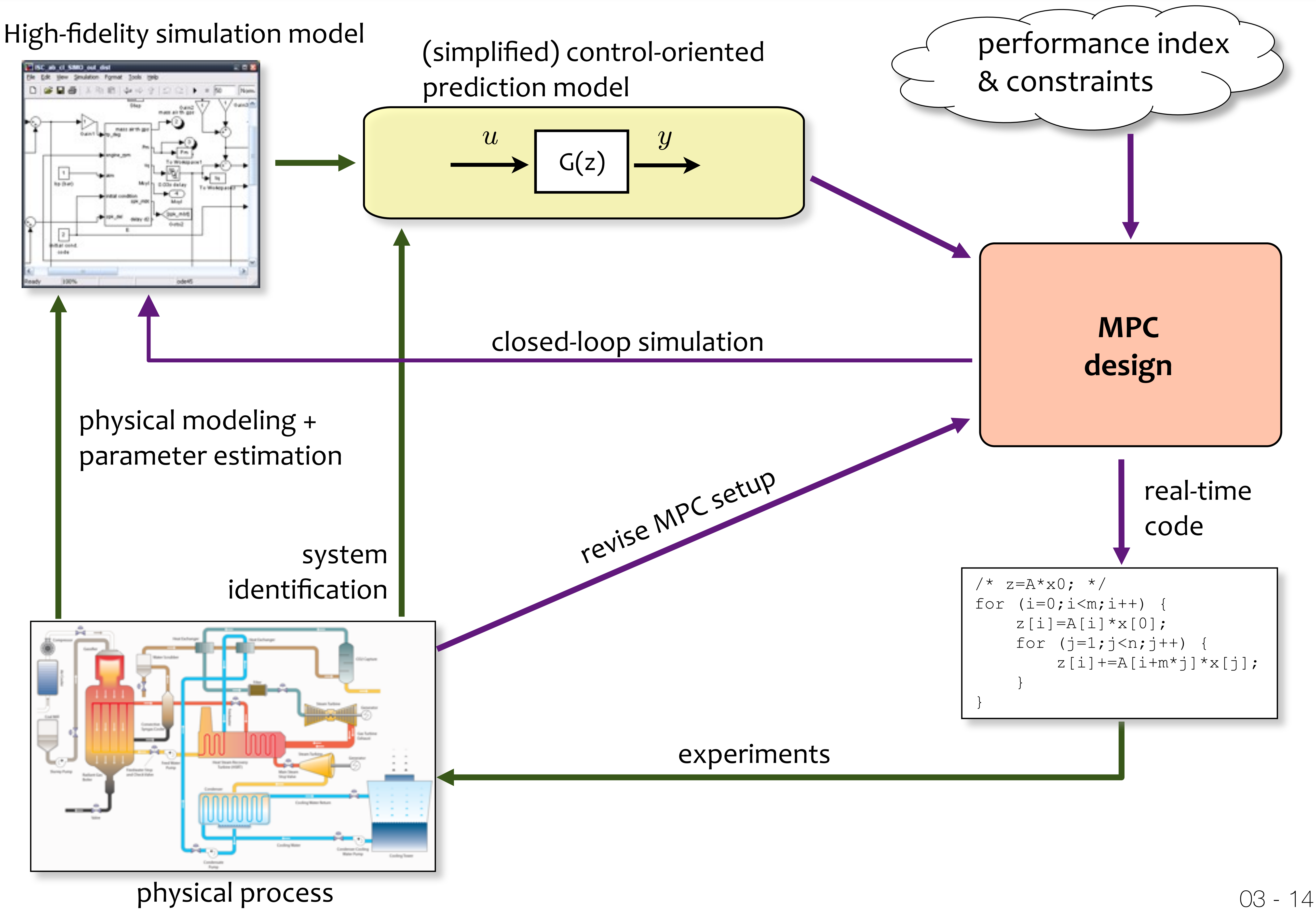


Automatically **operate a large-scale urban drinking water network**

Challenges: minimize network's operating costs and ensure demand satisfaction by controlling pumping in real-time, considering storage **dynamics**, **topology**, physical **constraints**, **stochastic** uncertainty (water demand, energy prices)

FP7-ICT project "**WIDE - Decentralized and Wireless Control of Large-Scale Systems**"
FP7-ICT project "**EFFINET - Efficient Integrated RT Monitoring & Control of Drinking Water Nets**"

MPC DESIGN FLOW



MPC TOOLBOXES

- **MPC Toolbox (The Mathworks, Inc.)**
  (Bemporad, Ricker, Morari, 1998-present)

  - Part of Mathworks' official toolbox distribution
  - Great for **education and research**

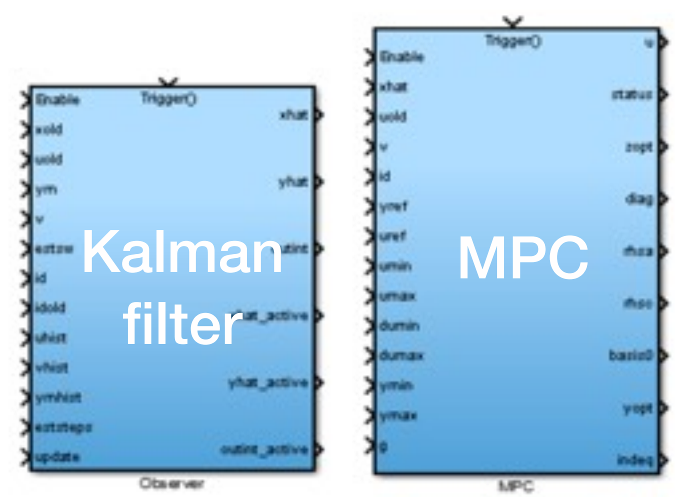
- **Hybrid Toolbox**
  (Bemporad, 2003-present)

> 6k downloads

  - Free download: http://cse.lab.imtlucca.it/~bemporad/hybrid/toolbox/
  - Great for **research and education**

- **ODYS Toolbox**
  (Bemporad, Bernardini, 2013-present)

  - Provides flexible and customized MPC control **design** and **seamless integration** in production systems
  - Real-time code written in plain C
  - **Designed for production**



ODYS
Advanced Controls & Optimization

PROS AND CONS OF MPC

✓ Extremely flexible control design approach:

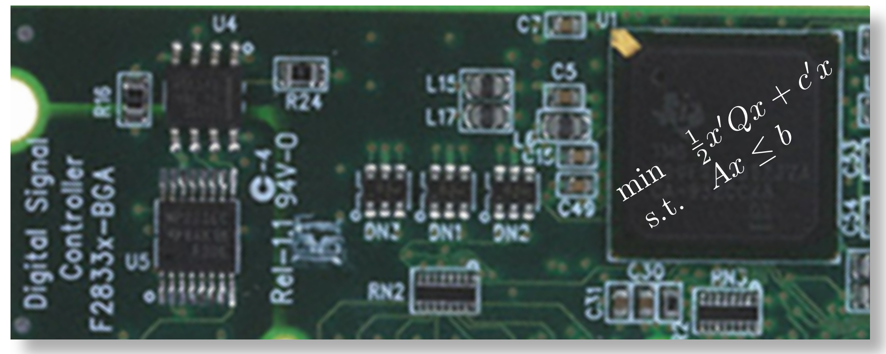
- Prediction model can be **multivariable**, w/**delays**, **time-varying**, w/ **disturbances**, ...
- Can exploit available **preview** on future references and measured disturbances
- Handles **constraints** on inputs and outputs
- Tuning similar to Linear Quadratic Regulator (**LQR**)

▸ Price to pay:

- Requires a (simple) **model** (experiments, systems identification, linearization)
- Many **degrees of freedom** (weights, horizons, constraints, ...)
- Requires **real-time computations** to solve the optimization problem

REQUIREMENTS FOR DEPLOYMENT OF MPC

embedded model-based optimizer



Requirements:

1. **Speed (throughput)**: solve optimization problem within sampling interval

2. **Robustness** (e.g., with respect to numerical errors)

3. Be able to run on **limited hardware** (e.g., 150 MHz) with **little memory**

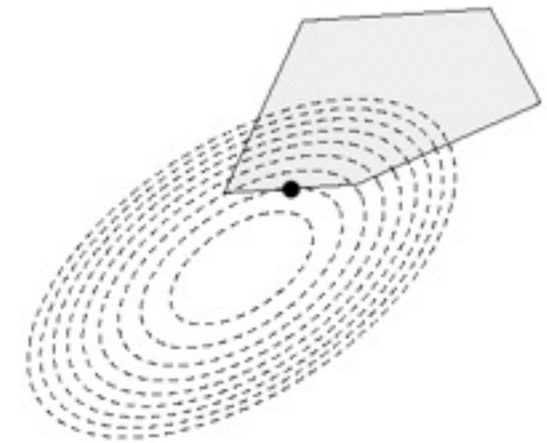
4. **Worst-case execution time** must be (tightly) estimated

5. **Code simple** enough to be validated/verified/certified
   (in general, it must be understandable by production engineers)

QUADRATIC PROGRAMMING

- Linear MPC requires solving a **Quadratic Program (QP)**

$$
\begin{aligned}
\min_{z} \quad & \frac{1}{2}z'Hz + x'(t)F'z + \frac{1}{2}x'(t)Yx(t) \\
\text{s.t.} \quad & Gz \leq W + Sx(t)
\end{aligned}
\qquad
z = \begin{bmatrix} u_0 \\ u_1 \\ \vdots \\ u_{N-1} \end{bmatrix}
$$



- Algorithms for QP have been studied since the 1950's ! (Beale, 1955)

A rich set of good QP algorithms is available today, and a lot of research is still going on !

FAST GRADIENT PROJECTION FOR (DUAL) QP

(Patrinos, Bemporad, IEEE TAC, 2014)

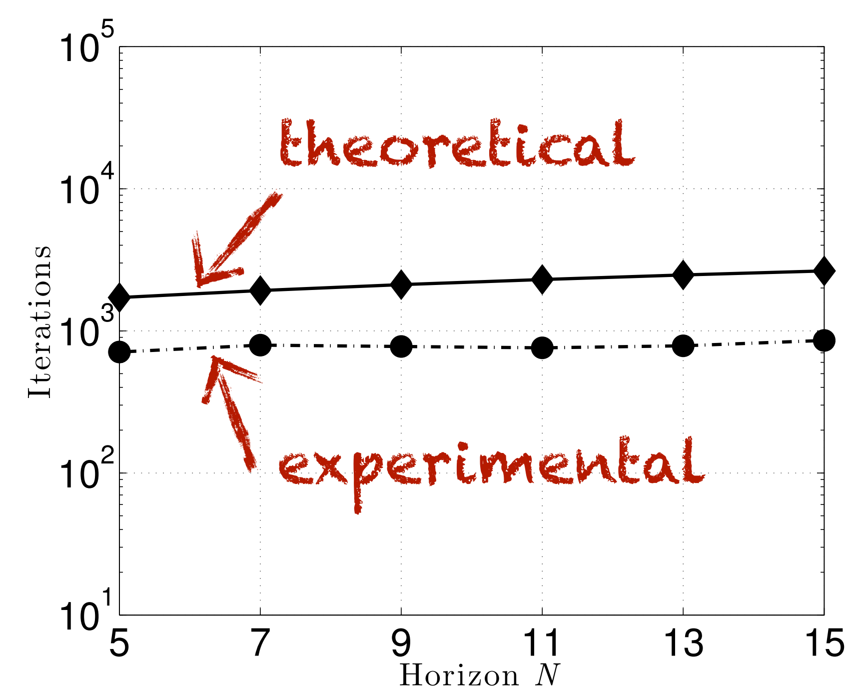
- Main on-line operations involve only **simple linear algebra**

```
while keepgoing && (i<maxiter),

    beta=(i-1)/(i-2).*(i>0);

    w=y+beta*(y-y0);
    z=-(iMG*w+iMc);
    s=GLz-bL;

    y0=y;

    % Check termination conditions
    if all(s<=epsGL),
        gapL=-w'*s;
        if gapL<=epsVL,
            return
        end
    end

    y=max(w+s,0);

    i=i+1;
end
```

- Convergence rate:

$$f(z_{k+1}) - f^* \leq \frac{2L}{(k+2)^2}\|z_0 - z^*\|^2$$

- Tight bounds on maximum number of iterations



- Can be used to warm-start other methods

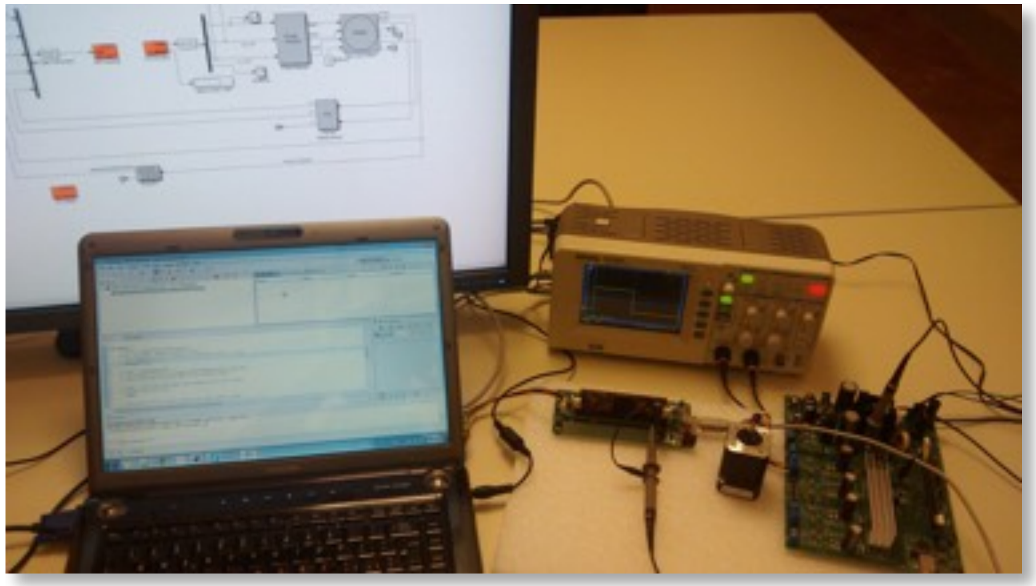
- Currently extended to mixed-integer problems

(Naik, Bemporad, work in progress)

EXPERIMENTS WITH EMBEDDED QP

TMS320F28335 controlCARD
(Real-time Control Applications)

- 32-bit Floating Point (IEEE-754);
- 150MHz clock;
- 68KB Ram / 512KB Flash.



var × constr.	GPAD	AS	ADMM	FBN
4×16	332 μs (18)	120 μs (3)	1.42 ms (62)	208 μs (2)
8×24	1.1 ms (22)	446 μs (5)	4 ms (77)	396 μs (2)
12×32	2.59 ms (27)	1.19 ms (7)	8.25 ms (82)	652 μs (2)

- **Active set** (AS) methods are usually the best on small problems:
  - excellent quality solutions within few iterations
  - less sensitive to preconditioning (= behavior is more predictable)
  - no need for advanced linear algebra packages

* GPAD = Dual Accelerated Gradient Projection (Patrinos, Bemporad, 2014)
* FBN = Forward-Backwards Netwon (proximal method) (Patrinos, Guiggiani, Bemporad, 2014)
* ADMM = Alternating Directions Method of Multipliers (Boyd et al., 2010)

MPC IN FINITE-PRECISION ARITHMETICS

(Patrinos, Guiggiani, Bemporad, 2013)

- Gradient projection works in fixed-point arithmetics

$$\max_i g_i(z_k) \leq \frac{2LD^2}{k+1} + L_v\epsilon_z^2 + 4D\epsilon_\xi$$

exponentially decreasing with number p of fractional bits

max constraint violation

Table 1
Fixed-point hardware implementation

Size [variables/constraints]	Time [ms]	Time per iteration [μs]	Code Size [KB]
10/20	22.9	226	15
20/40	52.9	867	17
40/80	544.9	3382	27
60/120	1519.8	7561	43

fixed point

Table 2
Floating-point hardware implementation

Size [variables/constraints]	Time [ms]	Time per iteration [μs]	Code Size [KB]
10/20	88.6	974	16
20/40	220.1	3608	21
40/80	2240	13099	40
60/120	5816	30450	73

floating point

32-bit Atmel SAM3X8E

ARM Cortex-M3 processing unit

84 MHz, 512 KB of flash memory and 100 KB of RAM

fixed-point about 4x faster than floating-point

LINEAR PARAMETER-VARYING (LPV) MPC

linear prediction model

$$\begin{cases} x_{k+1} = A(p(t))x_k + B_u(p(t))u_k + B_v(p(t))v_k \\ y_k = C(p(t))x_k + D_v(p(t))v_k \end{cases} \qquad x_0 = x(t)$$

- Weights, horizon, constraints can all depend on current parameter $p(t)$

$$\begin{array}{ll} \min_U & \frac{1}{2}U'H(p(t))U + x'(t)F(p(t))'U \\ \text{s.t.} & G(p(t))U \leq W(p(t)) + S(p(t))x(t) \end{array}$$

(convex) Quadratic Program (QP)

All QP matrices must be constructed on line

- Can be extended to LTV (Linear Time-Varying) prediction models
- LPV/LTV models can be obtained from linearization of nonlinear models or from black-box LPV system identification

LINEARIZATION AND TIME-DISCRETIZATION

- Model is nonlinear and continuous-time

$$\frac{dx}{dt} = f(x(t), u(t))$$

- Linearization around a nominal state $\bar{x}(t)$ and input $\bar{u}(t)$ (an **equilibrium**, a reference **trajectory**, or the **current** values)

$$\begin{aligned}\frac{dx}{dt}(t+\tau) \simeq & \left.\frac{\partial f}{\partial x}\right|_{\bar{x}(t),\bar{u}(t)} (x(t+\tau) - \bar{x}(t)) \\ & + \left.\frac{\partial f}{\partial u}\right|_{\bar{x}(t),\bar{u}(t)} (u(t+\tau) - \bar{u}(t)) + f(x(t), u(t))\end{aligned}$$

- Conversion to discrete-time linear prediction model

discrete-time LPV model →

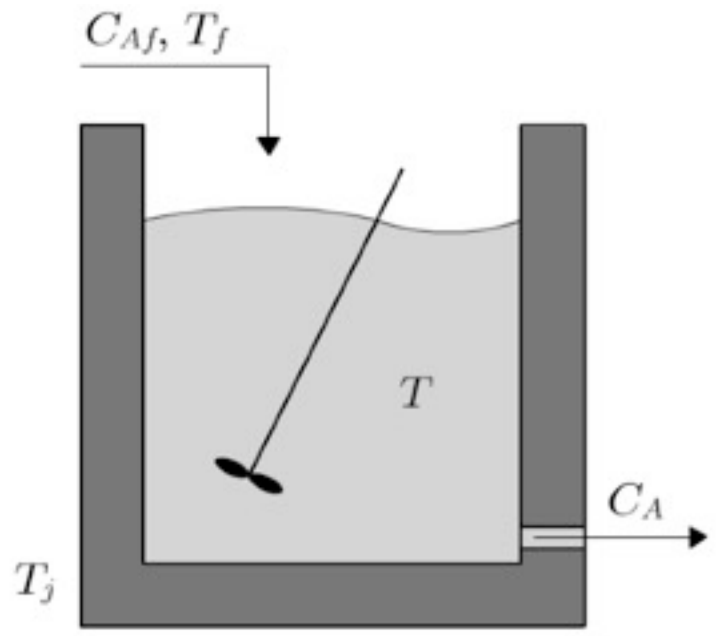
$$x_{k+1} = \underbrace{\left(I + T_s \left.\frac{\partial f}{\partial x}\right|_{\bar{x}(t),\bar{u}(t)}\right)}_{A(t)} x_k + \underbrace{\left(T_s \left.\frac{\partial f}{\partial u}\right|_{\bar{x}(t),\bar{u}(t)}\right)}_{B(t)} u_k + \underbrace{f_k}_{f(t)}$$

model matrices depend on current time t

EXAMPLE: LTV-MPC OF A NONLINEAR CSTR SYSTEM

- MPC control of a diabatic continuous stirred tank reactor (CSTR)

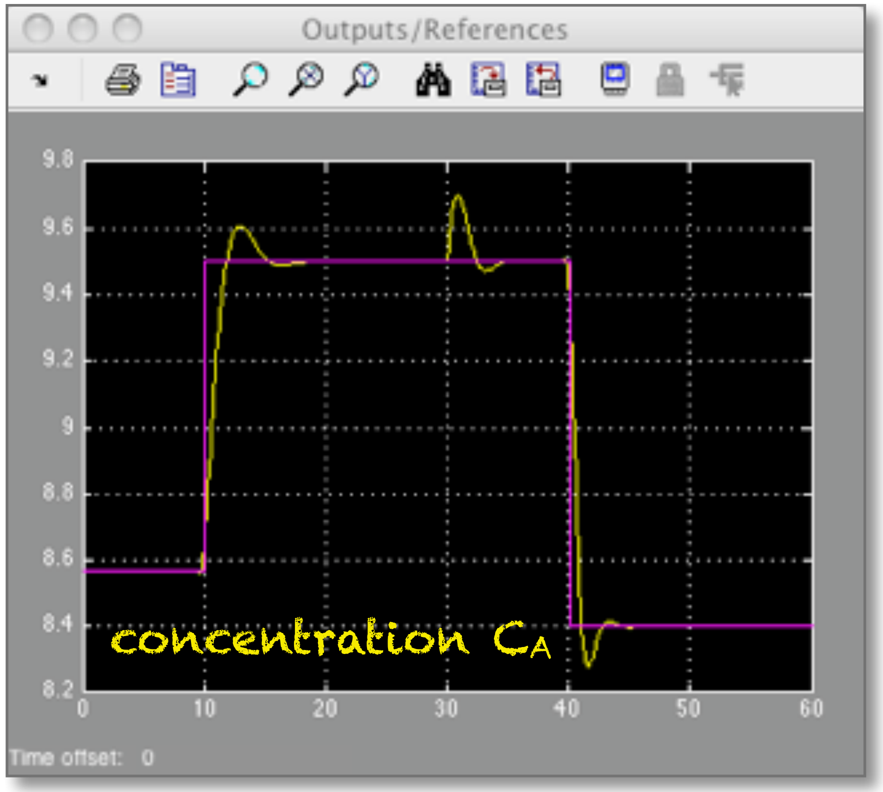
- Process model is rather nonlinear:



$$\frac{dC_A}{dt} = \frac{F}{V}(C_{Af} - C_A) - C_A k_0 e^{-\frac{\Delta E}{RT}}$$

$$\frac{dT}{dt} = \frac{F}{V}(T_f - T) + \frac{UA}{\rho C_p V}(T_j - T) - \frac{\Delta H}{\rho C_p} C_A k_0 e^{-\frac{\Delta E}{RT}}$$

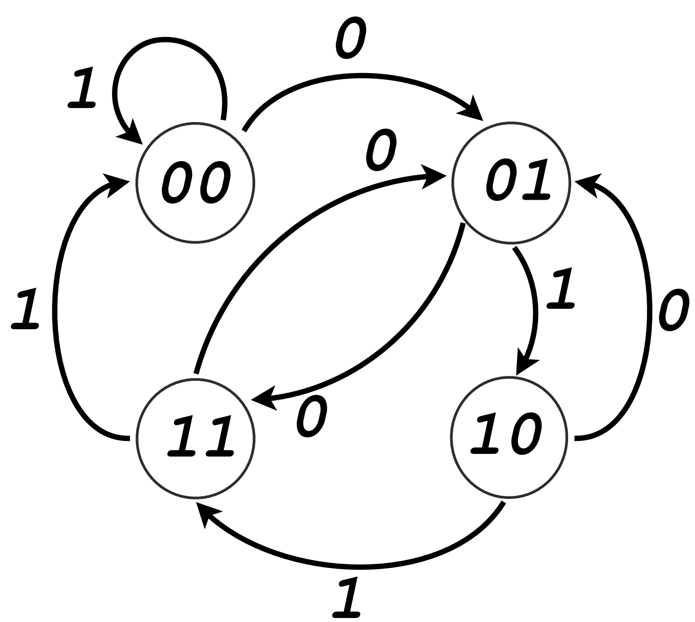
- T : temperature inside the reactor $[K]$ (state)
- C_A : concentration of the reagent in the reactor $[kgmol/m^3]$ (state)
- T_j : jacket temperature $[K]$ (input)
- T_f : feedstream temperature $[K]$ (measured disturbance)
- C_{Af} : feedstream concentration $[kgmol/m^3]$ (measured disturbance)



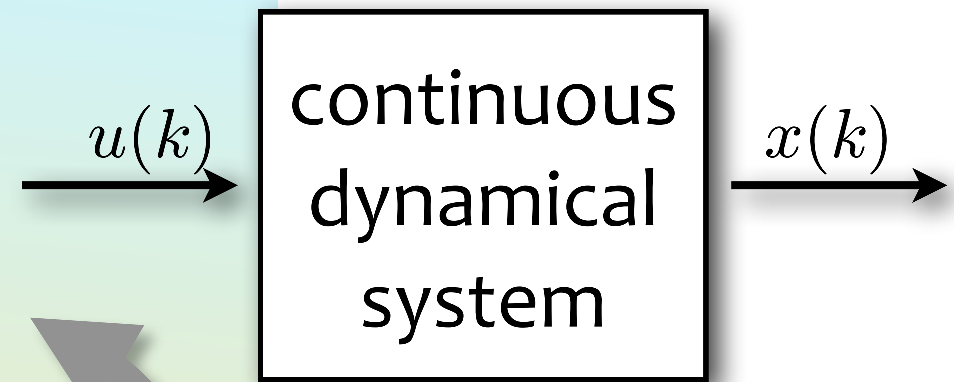
- Objective: manipulate T_j to regulate C_A on desired set-point

MODEL PREDICTIVE CONTROL OF HYBRID SYSTEMS

cyber system



physical system



hybrid dynamical system

- Variables are **discrete-valued**

  $x \in \{0,1\}^{n_b}, \; u \in \{0,1\}^{m_b}$

- Dynamics = **finite state machine**
- **Logic** constraints, **rules**

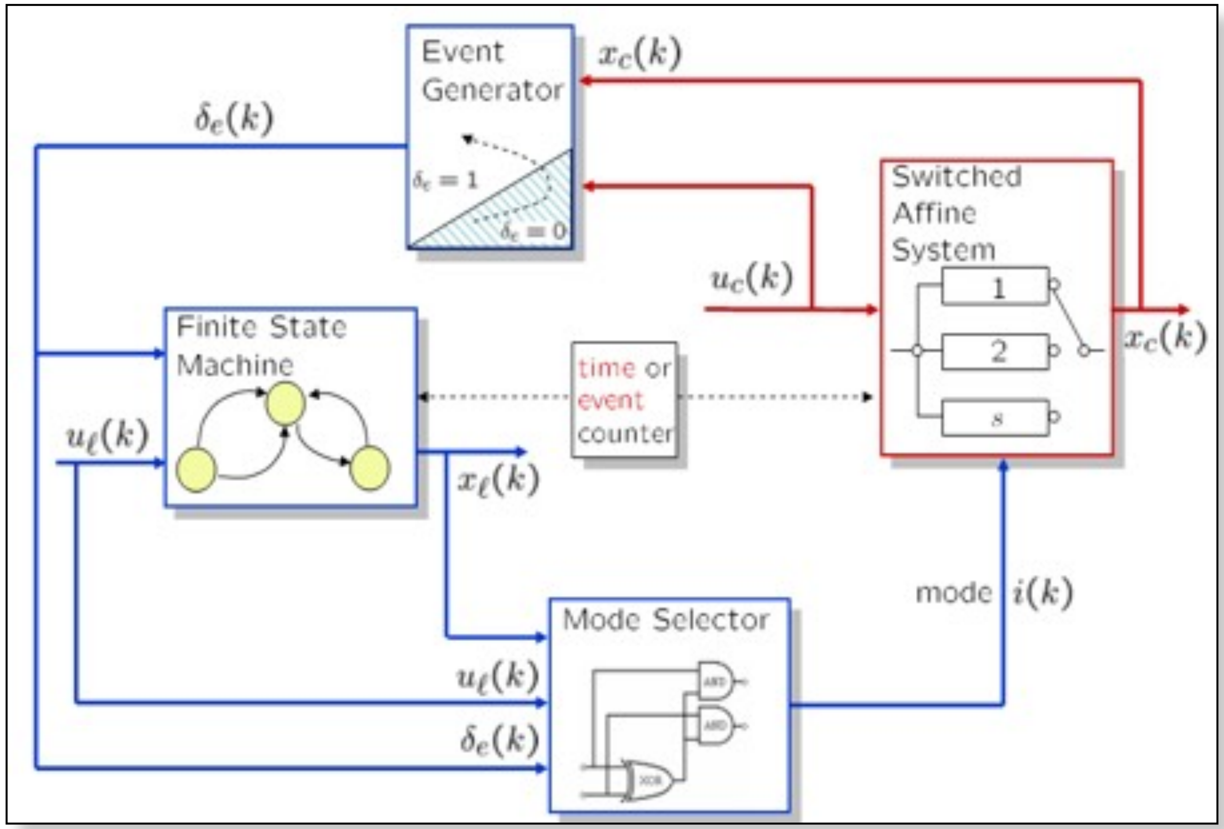
- Variables are **real-valued**

  $x \in \mathbb{R}^{n_c}, \; u \in \mathbb{R}^{m_c}$

- **Difference/differential equations**
- **Linear inequality** constraints

HYBRID MODEL PREDICTIVE CONTROL

Discrete Hybrid Automaton (Torrisi, Bemporad, 2004)



Hybrid Toolbox (Bemporad, 2003+)

http://cse.lab.imtlucca.it/~bemporad/hybrid/toolbox

Mixed Logical Dynamical (MLD) systems (Bemporad, Morari 1999)

$$\begin{cases} x_{k+1} = Ax_k + B_1 u_k + B_2\delta_k + B_3 z_k + B_5 \\ y_k = Cx_k + D_1 u_k + D_2\delta_k + D_3 z_k + D_5 \\ E_2\delta_k + E_3 z_k \leq E_4 x_k + E_1 u_k + E_5 \end{cases}$$

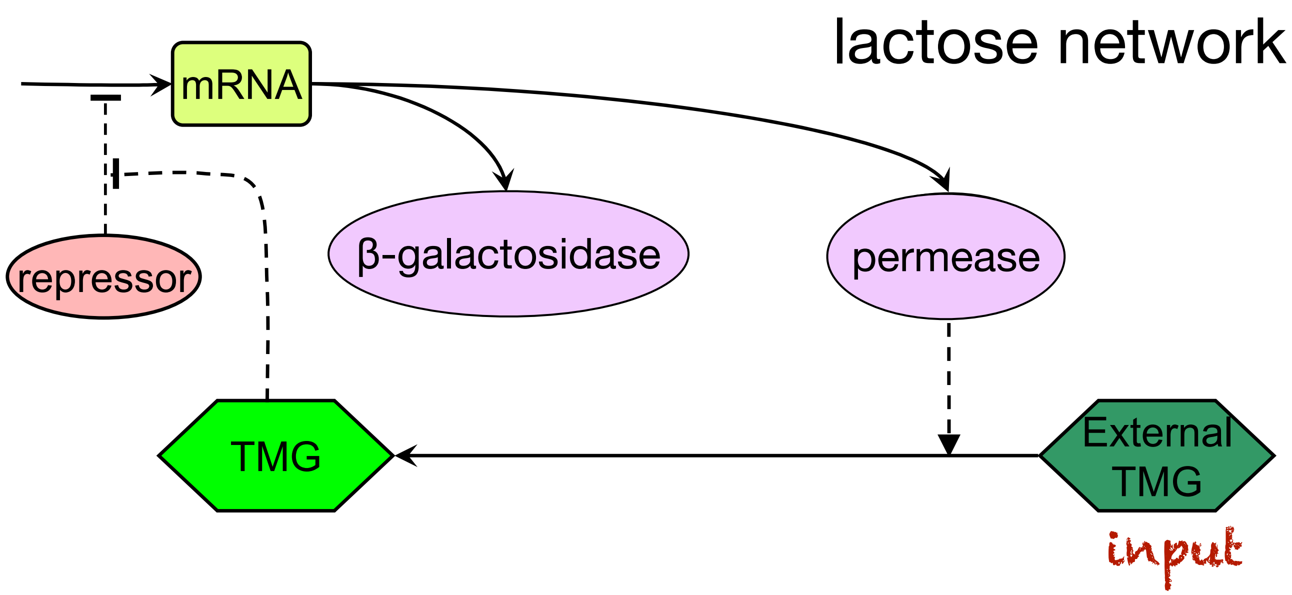
continuous & binary variables

- MPC problem can be solved by **mixed-integer programming (MIP)**
- Excellent public domain/commercial packages exist to solve MIP's

HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

(Julius, Sakar, Bemporad, Pappas, 2007)

- **Goal:** control the **lactose regulation system** of a colony of *E. coli*

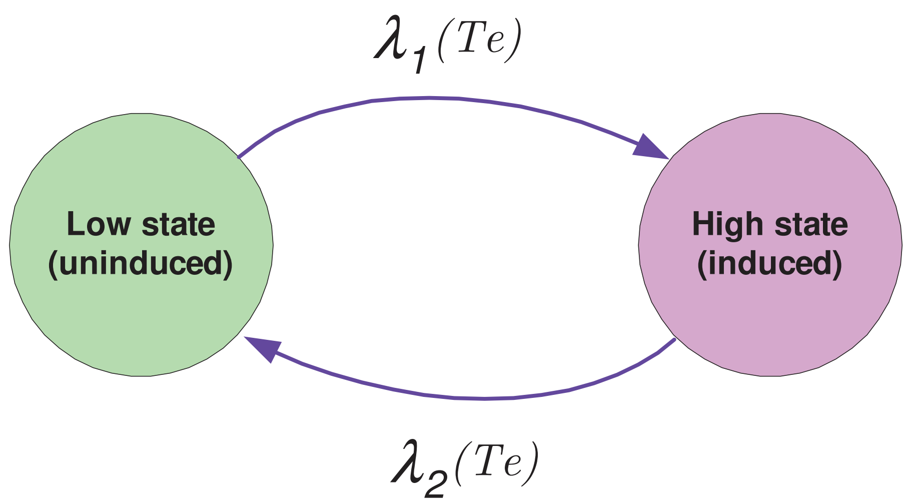


TMG = *thio-methyl galactosidase* concentration

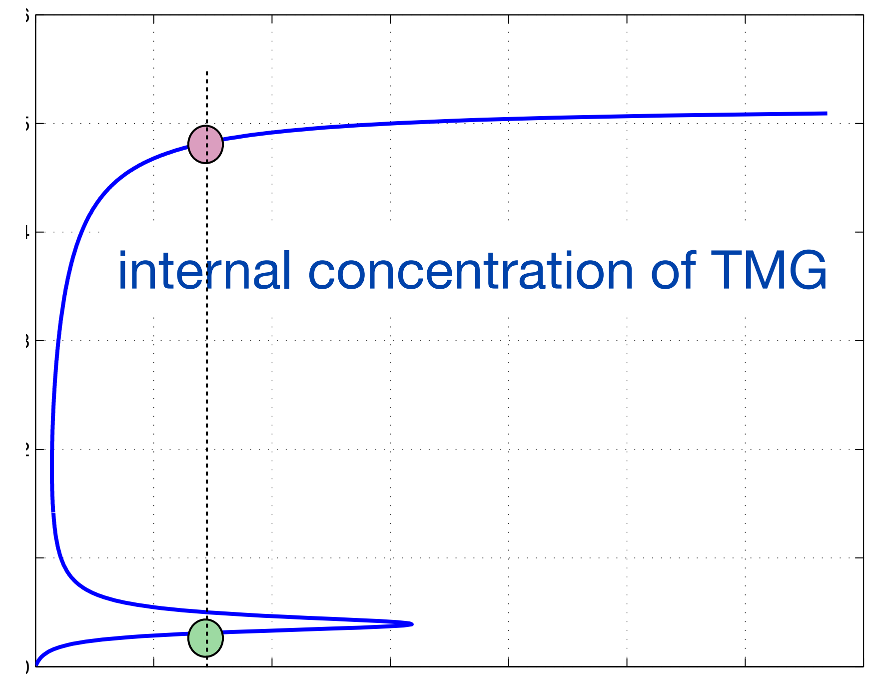
- Model, measurements, and actuation are at the **entire colony** level

HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

- **Bistable** lactose regulation system of E. coli



two-state Markov chain



Te = external concentration of TMG

- The probabilities x_{lo}, x_{hi} to be in low/high state satisfy the dynamics

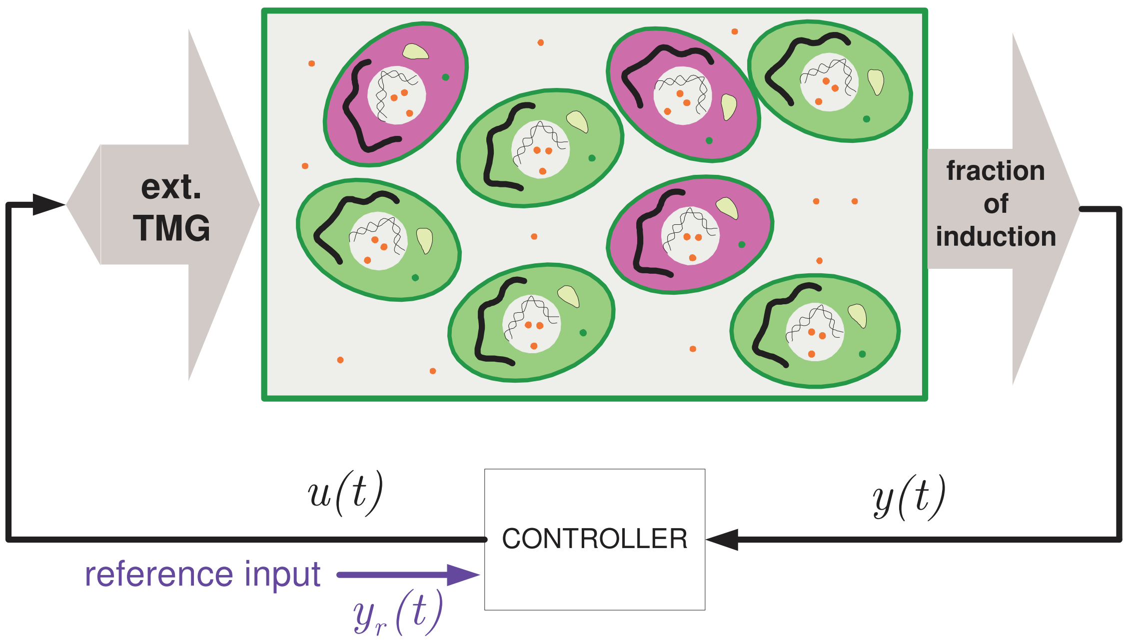
$$\frac{d}{dt}\begin{bmatrix} x_{\mathrm{lo}} \\ x_{\mathrm{hi}} \end{bmatrix} = \begin{bmatrix} -\lambda_1(T_e) & \lambda_2(T_e) \\ \lambda_1(T_e) & -\lambda_2(T_e) \end{bmatrix} \begin{bmatrix} x_{\mathrm{lo}} \\ x_{\mathrm{hi}} \end{bmatrix}$$

- Transition rates λ_1, λ_2 modeled as **piecewise constant** functions of T_e

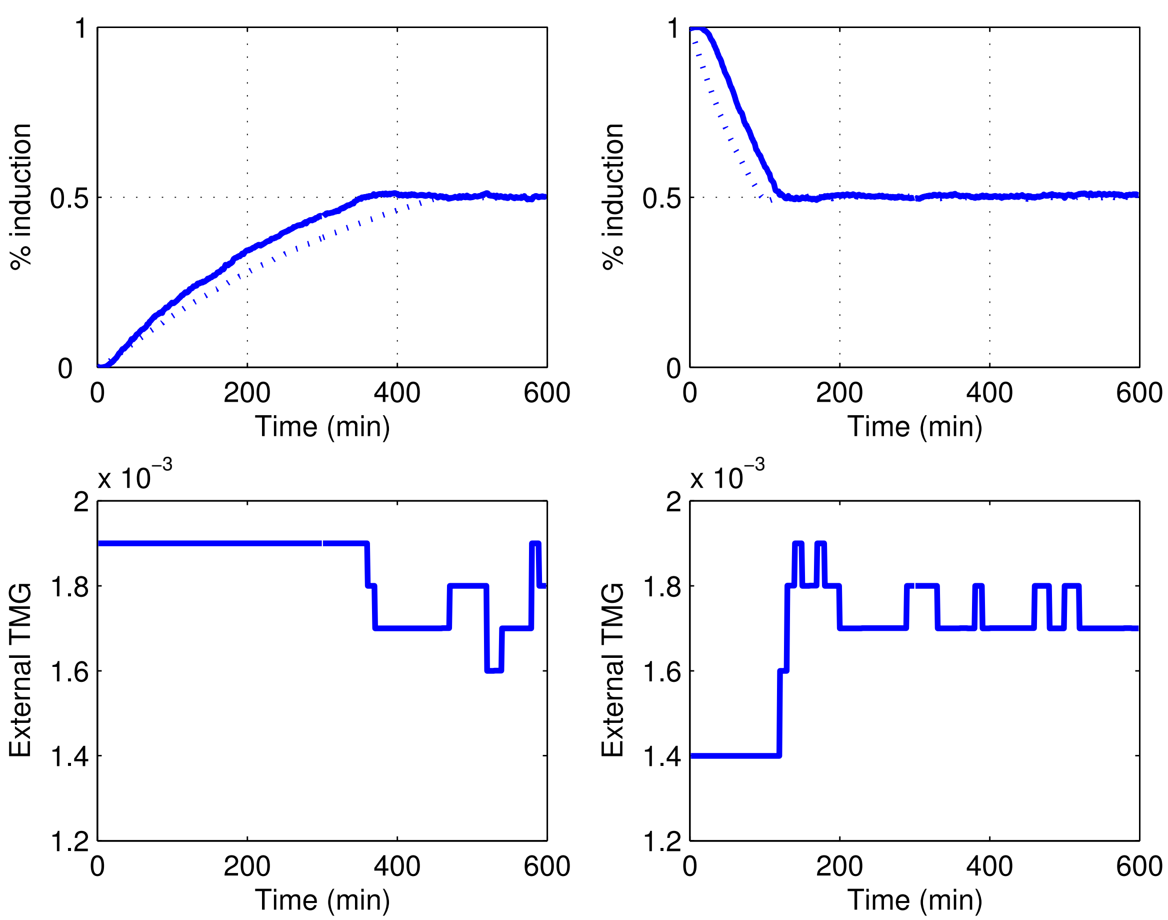
$T_e\,[10^{-3}\mathrm{mM}]$	$\lambda_1(T_e)\,[\mathrm{min}^{-1}]$	$\lambda_2(T_e)\,[\mathrm{min}^{-1}]$
$[1.4, 1.5)$	$8.68 \cdot 10^{-4}$	$5.91 \cdot 10^{-3}$
$[1.5, 1.6)$	$9.27 \cdot 10^{-4}$	$3.61 \cdot 10^{-3}$
$[1.6, 1.7)$	$1.13 \cdot 10^{-3}$	$2.36 \cdot 10^{-3}$
$[1.7, 1.8)$	$1.39 \cdot 10^{-3}$	$1.54 \cdot 10^{-3}$
$[1.8, 1.9)$	$1.67 \cdot 10^{-3}$	$9.53 \cdot 10^{-4}$
$[1.9, 2.0)$	$1.93 \cdot 10^{-3}$	$5.54 \cdot 10^{-4}$

HYBRID MPC OF INDUCTION OF ESCHERICHIA COLI

- Hybrid MPC problem
  - switched linear system
  - constraints on input T_e and dT_e/dt
  - penalties on tracking error $y-y_r$ and input rate dT_e/dt



- Closed-loop results
  - MPC controller developed with **Hybrid Toolbox** in MATLAB
  - Mixed-Integer Linear Program solver GLPK
  - solution time: 32 ms (worst case=**280 ms**) on 1.2 GHz laptop
  - sampling time = **10 min**



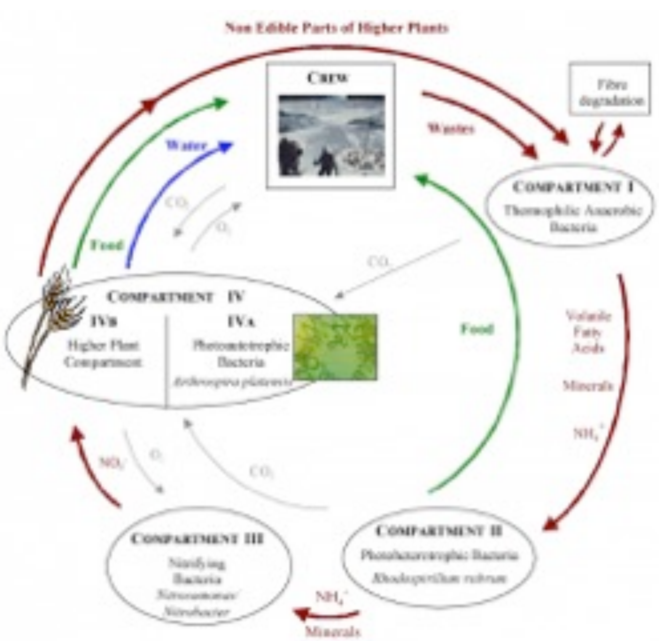
CONCLUSIONS

- MPC can easily handle **multivariable control** problems with **constraints** in an **optimized** way

- **Easy to design** and **reconfigure,** and to handle **uncertainty**

- Long history of success in the *process industries* now spreading to the *automotive industry* (and others)

- MATLAB design tools and production-ready C-code are available



Is MPC a good control technology for life-support systems ?

YES.

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